New firm creation and failure: a matching approach

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New Firm Creation and Failure: A Matching Approach

Thomas Gries*, Stefan Jungblut** and Wim Naudé#

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Abstract

We propose that the rate of creation and failure of new firm start-ups can be modelled as a search and matching process, as in labor market matching models. Deriving a novel Entrepreneurship-Beveridge curve, we show that a successful start-up depends on the efficiency with which entrepreneurial ability is matched with business opportunity, and outline a number of possible applications of this matching approach to formalize the economics of entrepreneurship.

JEL classifications: L26, M13,O10, O14
Keywords: Entrepreneurship, start-ups, labor market matching

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1 Introduction

Although there are many definitions of entrepreneurship, most suggest that it is about the discovery and exploitation of opportunities (Shane and Venkataraman, 2000). How entrepreneurs perceive opportunities, and utilize them to start-up new firms and businesses has spawned a large body of literature (e.g. Buenstorf, 2007; Casson and Wadeson, 2007; O'Fiet and Patel, 2008, Plummer et al., 2007 and Ucbasaran et al. 2008).

A feature of the start-up process noted in this literature is that while there is a large pool of latent entrepreneurs, many with highly developed human capital, only a small proportion of them succeed in starting up a firm. This has been explained with reference to human capital (e.g. Lazear, 2005) and the nature (and context) of opportunities that prospective entrepreneurs face (e.g. Blanchflower and Oswald, 1998).

In this paper we take these two ideas – the human capital of prospective entrepreneurs and the nature of opportunities – to propose a novel way of understanding start-ups[1]. Our aim is to make a modest theoretical contribution to the economics of entrepreneurship, and in this respect add to our previous theoretical contributions in this regard (see Gries and Naudé, 2010;2011). In particular, we borrow and adapt the concept of labor market matching from the field of labor economics, and apply it to describe start-ups as the outcome of a match between entrepreneurs with appropriate ability (human capital) and business opportunities. Common obstacles to start-ups, such as insufficient credit or inappropriate regulations can then be understood as frictions in the matching process.

In this paper we outline the core of our idea. That is, we explain the essence of start-ups as a matching process, and identify a number of research questions for further elaboration.

2 The Matching Approach

2.1 Intuitive Explanation

At any time in the economy there exist a number of opportunities for entrepreneurs to start-up successful firms. These are constantly evolving, and may spur both new firm start-ups as well as firms exits (churn). A useful way to model this situation is the matching approach. It has been applied to various fields in economics. Representative references for the labor market are Montgomery (1991), Mortensen and Pissarides (1999), Acemoglu and Shimer (2000) and Pissarides (2000).

With the matching approach we can explain empirical features of entrepreneurship such as constantly evolving start-up opportunities, a high exit rate of new start-ups and the development of heterogeneous business ideas / products/processes (i.e. innovation). Activities are described by a failure of present

[1] This idea was first proposed in a more rudimentary fashion by Gries and Naudé (2010; 2011). Here we elaborate the idea and propose its more general use in a variety of settings in formalizing the role of the entrepreneur in economic theory.

activities, a search for new opportunities, and the matching process that leads to new start-up firms. A match between start-up profiles (reflecting entrepreneurial ability) and the requirements of the market will result in a start-up. Since the efficiency of this matching reflects the efficiency of overcoming frictions and information and transaction costs, the efficiency of the matching process also reflects the quality of the institutional framework in this market.

2.2 Entrepreneurs

Our model distinguishes between *active entrepreneurs*, n and *latent entrepreneurs* u. Entrepreneurs are the creators and subsequent owners and managers of the firms in our model. In this sense our notion of entrepreneurs corresponds to the definition of entrepreneurship as the 'process of starting and continuing to expand new businesses' (Hart, 2003:5). As these firms come into being through the spotting and utilization of opportunities our notion is also consistent with Shane and Venkataraman (2000)'s definition of entrepreneurship as being concerned about the use of opportunities. A latent entrepreneur is a person who would prefer to be an entrepreneur and who considers seeking, or is actively seeking, an opportunity (Blanchflower et al. 2001). Around 25 percent of the labor force in OECD countries may be latent entrepreneurs (ibid. p.610). Given entrepreneurs and latent entrepreneurs represent the *total entrepreneurial potential* in the economy, E, and can be written as $E = n + u$.

2.3 Opportunities

Latent entrepreneurs search for opportunities to start up a new firm. We assume that available opportunities are exogenously given - i.e. opportunities exist independently of entrepreneurs. We denote the total number of potential start-up opportunities by Ω. At any time t, there are three types of start-up opportunity. First, there are already *taken* opportunities, which have resulted in a number of active entrepreneurs and their start-ups, n. Second, there are a number ω of *unrealized profitable opportunities* ready for the taking by an alert latent entrepreneur. And third, there are unrealised but *idle* (or yet unproductive) opportunities available, denoted by δ. These may be temporary or informal opportunities that are currently not profitable. People are often forced into these opportunities when they cannot obtain wage employment or spot a profitable opportunity as a result of either personal characteristics or external economic conditions. The total number of opportunities for a start-up firm can thus be written as

$$\Omega = n + \omega + \delta$$

2.4 A Start-Up as a Match

A start-up firm comes into being when the entrepreneur spots and utilizes an opportunity that matches their abilities and "business plan." The number of new start–up firms that result from such a matching is $\mathcal{M}$ per period. In aggregate, this matching rate – or start-up rate – will be determined by three factors. The first is the environment for doing business in the country. This environment, including the institutional framework of the economy, will determine how efficient the matching process is. For instance, an alert entrepreneur may spot a profitable opportunity, but may be prevented for utilizing it (i.e. from being "matched" to the opportunity). The overall matching efficiency in the economy is denoted μ. The second determinant of the matching (start-up) rate is the extent of unrealized profitable opportunities, denoted by ω. This reflects the fact that latent entrepreneurs are often said to be constrained by a lack of suitable or profitable opportunities. The third determinant of the matching (start-up) rate is the capability of the entrepreneur, specifically on how intense the latent entrepreneur may be searching for opportunities. We denote this search intensity η of latent entrepreneurs u as aggregate ηu. It has often been noted in the literature that the keenness and effort of latent entrepreneurs varies quite considerably. Also, as we show in the next section, there are costs involved in searching-the greater the search intensity, the higher the cost. Given these determinants of the matching (start-up) rate we can assume that the matching rate $\mathcal{M}$ can be written as:

$$\mathcal{M} = \mu M\left(\omega, \eta u\right),$$

where ηu denotes effective search efforts of entrepreneurs. Throughout the paper, we will assume that the rate of matches per entrepreneur and the rate of matches per opportunity depends on the ratio of opportunities to entrepreneurs only, but not on the size of the economy. This implies linear homogeneity of the matching function. In case of increasing or decreasing returns to scale in matching, the effectiveness of the matching process would vary according to the size of the economy. Although this might be reasonable to some extent, we think that this effect should not be expected to be systematic. Rather it is due to differences in the institutional and business environment of the economies, captured by differences in μ. Further, for computational simplicity we will model the matching-function as a Cobb-Douglas function:

$$\mathcal{M} = \mu\omega^{\beta}\left(\eta u\right)^{1-\beta}.$$

From this we can see that the probability of a successful new firm start-up is $\mu M/u = \mu m$.

2.5 Optimal Search and Investment Intensity, Matching, and Firm Failure

At the individual level, any potential entrepreneur i will have to make some search efforts described by the intensity η_i to spot and seize up a start-up opportunity. As we mentioned, such a search is costly. The search cost per unit of search effort is c_i.

Existing entrepreneurs will have to invest a certain effort ψ_i to ensure their firm's survival. The optimal search intensity to enter the market and the optimal investment intensity (effort) to stay in the market will be the result of maximizing entrepreneurs' net present value.[2] For simplicity we assume that entrepreneurs are identical and all entrepreneurial ventures yield the same expected profit (net of wages). The optimization problem of a representative entrepreneur needs to include two states: (i) the state of being a wage employed latent entrepreneur searching for opportunities to start a business, and (ii) the state of being an entrepreneur and trying to stay in business.

i) For the state of a wage employed *latent entrepreneur*, the net present *value* of *searching*, W_i, is given by wage income w_i minus search costs c_i times search intensity $\eta_i$plus the *extra* entrepreneurial income that can be expected if a successful opportunity is found and realized as a profitable new start-up firm. If we take V_i as the value of entrepreneurial income then the *extra* entrepreneurial income can be written on average as $\Delta = V - W$. This extra entrepreneurial income is not certain - the expected average extra income is Δ weighted by the probability of matching. In the previous subsection we established that the probability of matching is μm_i. Since individual efforts affect the matching probability $m_i(\eta_i)$ for a given discount rate r we obtain

$$rW_i = w_i - c_i\eta_i + \mu m_i(\eta_i)\Delta$$

ii) For the state of an *existing entrepreneur* actively working to keep the firm going, the net present *value* of *being an active entrepreneur* V_i is

$$rV_i = v_i - \gamma_i\psi_i - \phi_i(\psi_i)\Delta$$

Here the profits are v_i. In order to survive in the market, the entrepreneur would need to invest $\gamma_i\psi_i$ with effort ψ_i. These required investments reflect the transitory and dynamic nature of markets and existing institutional arrangements for the firm. Despite such investments, a firm failure can still occur. We denote the rate of firm failure by ϕ_i. From the perspective of the individual entrepreneur i, their investment efforts ψ_i may reduce the likelihood of firm failure ϕ_i which follows $\phi_i = \phi_i(\psi_i)$, $\phi_{\psi_i} := \partial\phi_i/\partial\psi_i < 0$, $\phi_{\psi_i\psi_i} := \partial^2\phi_i/\partial\psi_i^2 > 0$.

The above implies that the entrepreneur has the *choice to extend personal effort* to enhance the probability of finding a match, and to lower the probability of firm failure. They can maximize the expected income in both states of occupation, being a wage employed latent entrepreneur still searching for an opportunity, or being an active entrepreneur trying to stay in business. Thus the optimal search intensity, and the optimal effort to make the investments in the business most effective, will be a result of the following maximization exercise:

$$\begin{aligned}\max_{\eta_i} &\quad : \quad rW_i = w_i - c_i\eta_i + \mu m_i(\eta_i)\Delta \\ \max_{\psi_i} &\quad : \quad rV_i = v_i - \gamma_i\psi_i - \phi_i(\psi_i)\Delta\end{aligned}$$

[2] We can also introduce unemployed persons searching for opportunities while still on welfare benefits, but for the sake of tractability we abstract from this possibility for now.

From the F.O.C. we obtain an optimal search effort η^* and optimal investment effort ψ^* by using the implicit function theorem[3]

$$\eta^* = \eta^* (u, \omega, \Delta_i, c_i, \mu), \quad \text{with} \quad \begin{array}{l} \eta_u < 0, \quad \eta_\omega > 0, \quad \eta_\Delta > 0, \\ \eta_{c_i} < 0, \quad \eta_\mu > 0 \end{array}$$

$$\psi^* = \psi^* (\Delta, \gamma_i), \quad \text{with} \quad \psi_{\gamma_i} < 0, \qquad \psi_\Delta > 0$$

2.6 Aggregate Equilibrium Outcome

Assuming identical behavior across entrepreneurs we can now turn to consider the implications for the economy's aggregate equilibrium outcome.

First, we obtain the representative wealth differential Δ of the two wealth levels (W and V) associated with being a *latent entrepreneur* (searching for a start-up opportunity), or with being an *active entrepreneur (*trying to stay in the market). Defining the vector $x = (u, \omega, \Delta, c, \mu)$ we obtain an implicit relation for this wealth differential:

$$\Delta = \frac{v - w + c\eta (x) - \gamma\psi (\Delta, \gamma)}{r + \phi (\psi (\Delta, \gamma)) + \mu m (\eta (x))} \tag{1}$$

This equation determines the wealth differential Δ as the present value of the net income difference of the two states. The discount factor equals the interest rate r plus transition probabilities.

Second, we can consider differences in new firm start-ups and firm failures as describing the market dynamics for firm creation and failure in the economy. In the long-run stationary equilibrium the number of new firm start-ups will equal the number of firm failures. Given the probability of firm failure discussed in the previous subsection, the number of firm failures on the aggregate level is ϕn. The number of matched new firm start-ups is equal to μM. Hence the dynamics of firms is $\dot{n} = \mu M - \phi n$. The associated stationary flow equilibrium condition is:

$$\dot{n} = 0 \quad \Leftrightarrow \quad \mu M = \phi n \tag{2}$$

Third, in order to determine the aggregate equilibrium number of start-ups we also need to consider the dynamics of opportunities in the economy. We suppose that these dynamics are captured by two probabilities denoted p and q. Here p denotes the probability that profitable opportunities – either filled or vacant – become unprofitable, while q denotes the probability of formerly idle opportunities becoming profitable. These probabilities may be determined by exogenous changes including structural change, the rate and nature of economic growth, political instability, and technological progress. Thus the dynamics (rate of change) in idle start-up opportunities is $\dot{\delta} = p (\omega + n) - q\delta$. The associated stationary flow equilibrium for opportunities is[4]

[3] See appendix 1.
[4] We use the definition $\Omega = n + \omega + \delta$ to substitute for δ.

$$\dot{\delta} = 0 \quad \Leftrightarrow \quad \omega = \frac{q}{p+q}\Omega - E + u \tag{3}$$

With equation (1), (2) and (3) we obtain a system of three equations with three endogenous variables (u, ω, Δ). The system is determined by information, transactions costs, institutional features and general business environment. These are reflected by the general matching efficiency μ, transaction costs c in the start-up phase, and the adjustment costs γ for firm growth and survival. Furthermore the general business environment is reflected in the ability of markets to absorb new product variations Ω and the entrepreneurial potential of the economy E.

$$\begin{aligned}
0 &= F = \phi(\psi^*)(E-u) - \mu M(\omega, u, \eta^*) && \text{stationary matching equilibrium} \\
0 &= G = \Delta(r + \phi(\psi^*) + \mu M(\omega, u, \eta^*)/u) - v + \gamma\psi^* + w - c\eta^* && \text{wealth diff.} \\
0 &= H = \omega - \frac{q}{p+q}\Omega + E - u && \text{supply of profitable opportunities}
\end{aligned}$$

From this system of equations we can derive Proposition 1.

Proposition 1 *The economy [the system of equations F, G, H] has a stationary matching equlibrium solution of firm creation and firm failure, and hence a stationary number of latent entrepreneurs u^*, unrealized but profitable opportunities ω^* and a stationary differential of entrepreneurial and labor wealth Δ^*, as long as $\frac{q}{p+q}\Omega - E > 0 \rightarrow \omega > u$.*

$$u^* = u^*(x), \quad \omega^* = \omega^*(x), \quad \Delta^* = \Delta^*(x)$$

where $x = (\mu, c, p, q, \Omega, E, v, w)$.

Proof: See Appendix 2.

Proposition 1 states that we can find a constant number of firms in the economy. With a stationary number of firms we can identify to what extent the opportunities of this economy or the entrepreurial potential could be utilized. Further, we can also determine how high the stationary wealth premium Δ^* for a representative entrepreneur will eventually be. This type of information reflects the economy's efficiency with respect to entrepreneurial activities. In a perfect market economy without frictions all opportunities are seized and there is little need for an extra premium to become an entrepreneur. Therefore, we describe the market as a location (or institutional framework) which may or may not be fulfilling its purpose efficiently.

3 Implications and Comparative Statics

The main aim of our paper is to present the novel idea of explaining firm creation and failure as the outcome of a matching process. While we largely leave elaborations and the application of the model to future research, we want to illustrate how studying the comparative statics of the model can reveal the role of the various determinants of firm creation and failure, and can generate policy recommendations. In particular, we are interested in the start-up rate as the percentage rate of new firms in relation to existing firms ε, the survival rate as the percentage rate of successful surviving firms in relation to existing firms λ, and the total utilization of an economy's entrepreneurial potential defined as the percentage rate of existing firms in relation to the total entrepreneurial potential in the economy $\Psi = \frac{n}{E}$. For these central indicators we determine the effects of a) the general market environment and the institutional quality indicated by the matching efficiency parameter μ, b) information and transaction costs during the start-up phase c, and c) investment costs to keep the firm in the market γ. Finally we show that a growing economy promotes start-ups and firm survival, and generally improves the utilization of an economy's entrepreneurial potential. We state these ideas in Propositions 2, 3 and 4. All effects described in these propositions are also illustrated in figure 1.

Proposition 2 *An increasing matching efficiency, $d\mu > 0$, will (i) increase the matching and start-up rate in the economy $\varepsilon = \frac{\mathcal{M}}{n}$, (ii) decrease the rate of firm survival $\lambda = 1 - \phi$, and, (iii) improve the total utilization of an economy's entrepreneurial potential $\Psi = \frac{n}{E}$, as long as $\frac{q}{p+q}\Omega - E > 0 \rightarrow \omega > u$ (i.e. number of profitable opportunities is larger than number of latent entrepreneurs):*

$$\frac{d\varepsilon}{d\mu} > 0, \quad \frac{d\lambda}{d\mu} < 0, \quad \frac{d\Psi}{d\mu} > 0.$$

Proof: See Appendix 3.

The above proposition suggests the intuitively expected effects. However, the effect $\frac{d\lambda}{d\mu} < 0$ may require a short explanation. An increasing matching efficiency in the start-up phase increases the profitability of start-up efforts. Therefore, greater effort is invested in this activity than in staying in business. As a result, the start-up rate increases and the survival rate decreases.

Proposition 3 *Increasing information and transaction costs when starting-up $dc > 0$, will (i) reduce the matching and the start-up rate ε, (ii) increase the efforts of staying in business and hence the rate of firm survival λ, and (iii) reduce the total utilization of an economy's entrepreneurial potential Ψ. As long as $\frac{q}{p+q}\Omega - E > 0 \rightarrow \omega > u$.*

$$\frac{d\varepsilon}{dc} < 0, \quad \frac{d\lambda}{dc} > 0, \quad \frac{d\Psi}{dc} < 0.$$

Further, increasing investment costs for keeping the firm in the market $d\gamma > 0$, will (i) increase the efforts to start up a new firm and the start-up rate ε, (ii) reduce the rate of firm survival λ, and (iii) reduce the total utilization of an economy's entrepreneurial potential Ψ,

$$\frac{d\varepsilon}{d\gamma} > 0, \quad \frac{d\lambda}{d\gamma} < 0, \quad \frac{d\Psi}{d\gamma} < 0.$$

Proof: See Appendix 3.

While most of the effects described in the proposition are intuitively clear the cross-effects of the two kinds of cost require some explanation. If the transaction costs of starting a new firm increase ($dc > 0$) it will be relatively more attractive to stay in business, hence the relative effort t ensure survival increases and the survival rate rises. Symmetrically, if investment costs for keeping the firm in business increase ($d\gamma > 0$) it becomes relatively more attractive to potentially follow a new business idea and try something new rather than keeping the existing firm going. Hence less efforts are invested in firm survival since new ideas can be tried easily. As a result the survival rate decreases and the start-up rate rises.

Proposition 4 *A general economic expansion leading to a general increase in opportunities Ω will (i) increase the matching and the start-up rate ε, (ii) decrease firm survival λ, and (iii) improve the total utilization of an economy's entrepreneurial potential Ψ, as long as $0 < 1 - \frac{\overset{(-)}{\phi_\psi}}{\phi}\frac{\phi_{\psi_i}}{\phi_{\psi_i\psi_i}}$:*[5]

$$\frac{d\varepsilon}{d\Omega} > 0, \quad \frac{d\lambda}{d\Omega} < 0, \quad \frac{d\Psi}{d\Omega} > 0$$

Proof: See Appendix 4.

Figure 1 describes how the matching equilibrium can be endogenously determined. In figure 1 the *entrepreneurship Beveridge curve* describes the equilibrium relation between unrealized profitable opportunities and latent entrepreneurs trying to match their idea to an opportunity. Hence this relation indicates market efficiency in a potential equilibrium. For instance, an entrepreneurship Beveridge curve located more in the north-west of the figure indicates inefficiency: Even if there are a large numbers of unrealized profitable opportunities, many latent entrepreneurs will still not be able to match their ideas to a profitable opportunity. Hence, if the curve was in the north-west of the figure it would imply increasing inefficiency due to strong frictions in information and

[5] This condition is a sufficient condition and states that the external market environment must have a sufficiently strong effect on the probability of staying in business. That is, even if an entrepreneur puts more effort into staying in the market, this additional effort has limited effects and will not strongly improve the chances of survival. This condition is also sufficient to ensure the negative slope and normal reactions of the entrepreneurial Beveridge curve in figure 1.

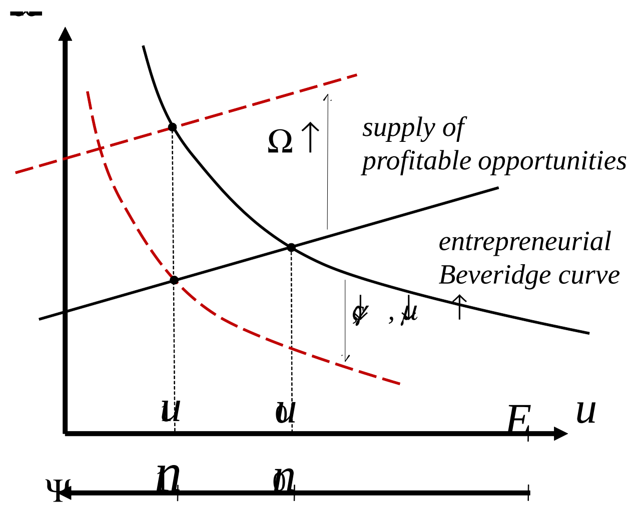

Figure 1: Entrepreneurial Beveridge curve and equlibrium start-up

transaction efficiency, or ineffective institutions, or strongly diverging interests between customers and potential entrepreneurs in the market. The slope of the entrepreneurship Beveridge curve describes how for given market conditions decreasing business opportunities drive down the number of firms in the market n and/or drive up the number of latent entrepreneurs u in the economy.

The second curve in figure 1 is the *supply curve of profitable opportunities*. This curve describes the relationship between latent entrepreneurs and the supply of profitable opportunities for the given in- and outflow connected with the *idle* (or yet unproductive) opportunities δ. Equilibrium in the market occurs where the Entrepreneurship Beveridge curve intersects the supply curve of profitable opportunities. While figure 1 enables us to graphically illustrate the matching equilibrium and comparative static adjustments, we can also graphically illustrate how changes in the market matching process affect important economic indicators like the utilization rate of entrepreneurial potential. For this purpose we can draw a second axis starting at the given number of potential entrepreneurs E. This axis points to the opposite direction than the u-axis because it counts the number of active entrepreneurs. For a given E this axis hence also indicates the utilization rate of the entrepreneurial capacity of this economy.

4 Concluding Remarks

The purpose of this paper is to offer a novel way of formalizing the processes of firm creation and failure in an endogenous growth model setting. This is done by considering successful start-ups as the result of a match between entrepreneurs and opportunities. In this matching process and the subsequent survival of new start-ups both entrepreneurial ability and search intensity and investment effort are significant. However, even when individual entrepreneurs raise their search intensity and investment efforts, firm start-up and failure rates will be affected by institutions and the conditions for doing business. Even though some entrepreneurs may overcome adverse conditions for doing business, many others will not, and the aggregate utilization of entrepreneurial capacity in the economy will be lower. Using a few comparative statics, we have illustrated how high costs of information and transactions, adjustment and investment in a changing market environment and deteriorating conditions for doing business will decrease the matching (start-up) rate and increase the rate of firm failure. The policy implications are that measures to increase the aggregate utilization of entrepreneurial capacity in the economy need to address both the individual entrepreneur as well as the aggregate business environment. Business environment reform (BER), the core of most private sector development (PSD) programmes is clearly not enough.

If the creation and survival of new firm start-ups are an essential ingredient of economic growth and development process, then our approach offers a useful insight into the process underlying this churning of firms. Unlike other approaches, where firm start-ups are a function of a myriad of often weakly justified factors, factors that are treated separately from the determinants of firm failure, the matching approach treats both the creation and the failure of new start-ups essentially as a result of a mismatch between opportunities and entrepreneurs - including their ability and external environment.

The model could be further elaborated to include the linkages between search intensity and the degree to which entrepreneurship is valued in itself, as opposed to merely being an instrument to achieve other outcomes. Future research could investigate institutional entrepreneurship, that is to say, how individual search efforts could include efforts to change the business environment. Finally, attention could also be given to considering how innovation and innovation policy can assist in matching different types of entrepreneurs to specific opportunities, and to gaining a better understanding of how investors and researchers can be matched to venture capitalists. We believe these are just a few of the potential areas where labor economics' idea of matching can be applied to the formalization of entrepreneurship in economic theory.

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5 Appendices

5.1 Appendix 1: Determining Optimal Effort Levels

Search effort: Determining the optimal search effort, the effort function η and derivatives:

$$\max_{\eta_i} : rW_i = w_i - c_i\eta_i + \mu\frac{\omega^\beta (\eta_i u)^{1-\beta}}{u}\Delta$$

F.O.C. and S.O.C.:

$$0 = -c_i + \mu\frac{\Delta}{u}(1-\beta)\frac{M}{\eta_i}, \qquad 0 > -\mu\frac{\Delta}{u}(1-\beta)\beta\omega^\beta\eta_i^{-\beta-1}u^{1-\beta}$$

Optimal search effort of each entrepreneur is determined by using the implicit function theorem from the F.O.C. and S.O.C. We obtain

$$\eta^* = \eta^*(u, \omega, \Delta, c_i, \mu), \quad \eta_\Delta > 0, \quad \eta_\omega > 0, \quad \eta_u < 0, \quad \eta_{c_i} < 0, \quad \eta_\mu > 0$$

Derivatives of the optimal effort:

$$\begin{aligned} \eta_\Delta &= \frac{\eta_i}{\Delta\beta} > 0, \quad \eta_\omega = \frac{\eta_i}{\omega} > 0, \quad \eta_u = -\frac{\eta_i}{u} < 0, \\ \eta_{c_i} &= \frac{-1}{\mu\frac{\Delta}{u}(1-\beta)\beta\frac{M}{\eta_i^2}} < 0, \quad \eta_\mu = \frac{\eta_i}{\mu\beta} > 0 \end{aligned}$$

Stay in market effort: Determining optimal effort to stay in the market, effort function ψ_i and derivatives:

$$\max_{\psi_i} : rV_i = v_i - \gamma_i\psi_i - \phi_i(\psi_i)\Delta$$

F.O.C. and S.O.C.

$$-\gamma_i - \phi_{\psi_i}\Delta = 0, \qquad -\phi_{\psi_i\psi_i}\Delta < 0$$

where $\phi_{\psi_i} := \partial\phi_i/\partial\psi_i$. From the f.o.c. and s.o.c. we obtain the optimal strategy

$$\psi^* = \psi^*(\Delta, \gamma_i)$$

with

$$\frac{\partial\psi_i}{\partial\gamma_i} =: \psi_{\gamma_i} = -\frac{1}{\phi_{\psi_i\psi_i}\Delta} < 0, \qquad \frac{\partial\psi_i}{\partial\Delta} =: \psi_\Delta = -\frac{\phi_{\psi_i}}{\phi_{\psi_i\psi_i}\Delta} > 0$$

5.2 Appendix 2: Proof of Proposition 1

Equations F, G, H [(1), (2), (3)] have continuous partial derivatives with respect to all variables. As all variables are positive, and since $\frac{q}{p+q}\Omega - E > 0 \rightarrow$

$\omega > u$, the determinant of the Jacobian matrix for the smooth function $f(x,y) = (F,G,H)(x,y)$, $y = (\omega, u, \Delta)$, $x = (\mu, c, q, p, \Omega, E, w)$ does not vanish:

$$A = \begin{pmatrix} -\mu\frac{M}{\omega} & -\phi & \overset{-}{\phi_\psi}\overset{+}{\psi_\Delta}(E-u) - \mu(1-\beta)\frac{M}{\Delta\beta} \\ \Delta\mu\beta\frac{m}{\omega} & -\Delta\mu\beta\frac{m}{u} & (r+\phi+\mu m) \\ 1 & -1 & 0 \end{pmatrix}$$

$$|A| = -(r+\phi+\mu m)\left(\phi+\mu\frac{M}{\omega}\right)+\mu\left(\overset{-}{\phi_\psi}\overset{+}{\psi_\Delta}(E-u)\Delta\beta - \mu(1-\beta)M\right)\underbrace{\left(\frac{m}{u}-\frac{m}{\omega}\right)}_{>0} \neq 0$$

So that the Jacobian matrix is invertible and the implicit function theorem can be applied. System [(1), (2), (3)] implicitly defines the functions

$$\begin{aligned} u^* &= u^*(\mu, c, v, q, p, \Omega, E, v, w) \\ \omega^* &= \omega^*(\mu, c, v, q, p, \Omega, E, v, w) \\ \Delta^* &= \Delta^*(\mu, c, v, q, p, \Omega, E, v, w). \end{aligned}$$

Comparative statics for the system F,G,H can be performed by taking the partial reaction from $Ada = dB$, with

$$\begin{aligned} da &= (d\omega, du, d\Delta)', \\ A &= \begin{pmatrix} -\mu\frac{M}{\omega} & -\phi & \overset{-}{\phi_\psi}\overset{+}{\psi_\Delta}(E-u) - \mu(1-\beta)\frac{M}{\Delta\beta} \\ \Delta\mu\beta\frac{m}{\omega} & -\Delta\mu\beta\frac{m}{u} & (r+\phi+\mu m) \\ 1 & -1 & 0 \end{pmatrix} \\ dB &= \begin{pmatrix} \frac{M}{\beta}d\mu - \frac{\eta u}{\Delta\beta}dc - (E-u)\phi_\psi\psi_\gamma d\gamma \\ 0 \\ \frac{q}{p+q}d\Omega - dE \end{pmatrix} \end{aligned}$$

5.2.1 Discussion of the Beveridge Curve:

From the first two rows of this system we obtain the entrepreneurial start-up Beveridge curve. The start-up Beveridge curve is in analogy to the labor market Beveridge curve.

Total differential for F:

$$\begin{aligned} 0 = & -\mu\frac{M}{\omega}d\omega - \phi du + \left((E-u)\phi_\psi\psi_\Delta - \mu(1-\beta)\frac{M}{\Delta\beta}\right)d\Delta - \frac{M}{\beta}d\mu + \frac{u\eta}{\Delta\beta}dc \\ & +(E-u)\phi_\psi\psi_\gamma d\gamma \end{aligned}$$

Total differential for G:

$$0 = \Delta\mu\beta\frac{m}{\omega}d\omega - \beta\Delta\mu\frac{m}{u}du + (r+\phi+\mu m)\,d\Delta$$

Plug in F:

$$\begin{aligned}
0 &= -\mu\frac{M}{\omega}d\omega - \phi du + \left((E-u)\phi_{\psi}\psi_{\Delta} - \mu\left(1-\beta\right)\frac{M}{\Delta\beta}\right)d\Delta - \frac{M}{\beta}d\mu + \frac{u\eta}{\Delta\beta}dc \\
&\quad +(E-u)\phi_{\psi}\psi_{\gamma}d\gamma \\
0 &= -\mu\frac{M}{\omega}d\omega - \phi du + \left((E-u)\phi_{\psi}\psi_{\Delta} - \mu\left(1-\beta\right)\frac{M}{\Delta\beta}\right) \\
&\quad \frac{\Delta\mu\beta}{-\left(r+\phi+\mu m\right)}\left(\frac{m}{\omega}d\omega - \frac{m}{u}du\right) - \frac{M}{\beta}d\mu + \frac{u\eta}{\Delta\beta}dc + (E-u)\phi_{\psi}\psi_{\gamma}d\gamma
\end{aligned}$$

Slope of the Beveridge curve: $\frac{d\omega}{du}$

$$\begin{aligned}
0 &= \left(r+\phi+\mu m\right)\mu\frac{M}{\omega}d\omega + \left(r+\phi+\mu m\right)\phi du \\
&\quad + \left((E-u)\phi_{\psi}\psi_{\Delta} - \mu\left(1-\beta\right)\frac{M}{\Delta\beta}\right)\Delta\mu\beta\left(\frac{m}{\omega}d\omega - \frac{m}{u}du\right)
\end{aligned}$$

$$\begin{aligned}
\frac{d\omega}{du} &= \frac{-[\left(r+\phi+\mu m\right)\phi - \left((E-u)\phi_{\psi}\psi_{\Delta} - \mu\left(1-\beta\right)\frac{M}{\Delta\beta}\right)\Delta\mu\beta\frac{m}{u}]}{[\left(r+\phi+\mu m\right)\mu\frac{M}{\omega} + \left((E-u)\phi_{\psi}\psi_{\Delta} - \mu\left(1-\beta\right)\frac{M}{\Delta\beta}\right)\Delta\mu\beta\frac{m}{\omega}]} \\
\frac{d\omega}{du} &= \frac{\left(\left(r+\phi+\mu m\right)\phi - \left((E-u)\phi_{\psi}\psi_{\Delta} - \mu\left(1-\beta\right)\frac{M}{\Delta\beta}\right)\Delta\mu\beta\frac{m}{u}\right)}{-\left(r+\phi+\beta m\mu + (\frac{E}{u}-1)\phi_{\psi}\psi_{\Delta}\Delta\beta\right)u\mu\frac{m}{\omega}}
\end{aligned}$$

$$for \qquad 0 < 1 - \frac{\overset{(-)}{\phi_{\psi}}}{\phi}\frac{\phi_{\psi_i}}{\phi_{\psi_i\psi_i}} \qquad \text{as sufficient condition.}$$

Location of the Beveridge curve: $\frac{d\omega}{d\gamma}, \frac{d\omega}{d\mu}, \frac{d\omega}{dc}$ **for** $0 < \mu m - \frac{\phi_{\psi_i}^2}{\phi_{\psi_i\psi_i}}\frac{n}{u}$:

$$\begin{aligned}
0 &= -\mu\frac{M}{\omega}d\omega - \phi du + \left((E-u)\phi_{\psi}\psi_{\Delta} - \mu\left(1-\beta\right)\frac{M}{\Delta\beta}\right)\left(\frac{\Delta\mu\beta}{-\left(r+\phi+\mu m\right)}\left(\frac{m}{\omega}d\omega - \frac{m}{u}du\right)\right) \\
&\quad -\frac{M}{\beta}d\mu + \frac{u\eta}{\Delta\beta}dc + (E-u)\phi_{\psi}\psi_{\gamma}d\gamma,
\end{aligned}$$

$$\frac{d\omega}{d\gamma} = \frac{\left(r+\phi+\mu m\right)(E-u)\phi_{\psi}\psi_{\gamma}}{\left(r+\phi+\beta m\mu + (\frac{E}{u}-1)\phi_{\psi}\psi_{\Delta}\Delta\beta\right)u\mu\frac{m}{\omega}} > 0$$

$$\frac{d\omega}{dc} = \frac{\left(r+\phi+\mu m\right)\frac{u\eta}{\Delta\beta}}{\left(r+\phi+\beta m\mu + (\frac{E}{u}-1)\phi_{\psi}\psi_{\Delta}\Delta\beta\right)u\mu\frac{m}{\omega}} > 0$$

$$\frac{d\omega}{d\mu} = -\frac{\frac{M}{\beta}\left(r+\phi+\mu m\right)}{\left(r+\phi+\beta m\mu + (\frac{E}{u}-1)\phi_{\psi}\psi_{\Delta}\Delta\beta\right)u\mu\frac{m}{\omega}} < 0$$

Total differential for H:

$$0 = d\omega - \frac{q}{p+q}d\Omega + dE - du$$

Appendix 3: Proof of Propositions 2, 3 and 4

$$
\begin{aligned}
da &= (d\omega, du, d\Delta)', \\
A &= \begin{pmatrix} -\mu\frac{M}{\omega} & -\phi & \overset{-}{\phi_\psi}\overset{+}{\psi_\Delta}(E-u)-\mu(1-\beta)\frac{M}{\Delta\beta} \\ \Delta\mu\beta\frac{m}{\omega} & -\Delta\mu\beta\frac{m}{u} & (r+\phi+\mu m) \\ 1 & -1 & 0 \end{pmatrix} \\
dB &= \begin{pmatrix} \frac{M}{\beta}d\mu-\frac{\eta u}{\Delta\beta}dc-(E-u)\phi_\psi\psi_\gamma d\gamma \\ 0 \\ \frac{q}{p+q}d\Omega-dE \end{pmatrix}
\end{aligned}
$$

solving for the four effects of c, γ, μ, and Ω on the number of latent entrepreneurs yields:

a) Effects on latent entrepreneurs:

$$
\frac{du^*}{dc}=\frac{-\frac{\eta u}{\Delta\beta}}{-\left[\left(\mu\frac{M}{\omega}+\phi\right)-\left(\overset{-}{\phi_\psi}\overset{+}{\psi_\Delta}(E-u)-\mu(1-\beta)\frac{M}{\Delta\beta}\right)\frac{\left(\overset{>0}{\frac{m}{u}-\frac{m}{\omega}}\right)}{(r+\phi+\mu m)}\Delta\mu\beta\right]}>0
$$

$$
\frac{du^*}{d\gamma}=\frac{-(E-u)\phi_\psi\psi_\gamma}{-\left[\left(\mu\frac{M}{\omega}+\phi\right)-\left(\overset{-}{\phi_\psi}\overset{+}{\psi_\Delta}(E-u)-\mu(1-\beta)\frac{M}{\Delta\beta}\right)\frac{\left(\overset{>0}{\frac{m}{u}-\frac{m}{\omega}}\right)}{(r+\phi+\mu m)}\Delta\mu\beta\right]}>0
$$

$$
\frac{du^*}{d\mu}=\frac{\frac{M}{\beta}}{-\left[\left(\mu\frac{M}{\omega}+\phi\right)-\left(\overset{-}{\phi_\psi}\overset{+}{\psi_\Delta}(E-u)-\mu(1-\beta)\frac{M}{\Delta\beta}\right)\frac{\left(\overset{>0}{\frac{m}{u}-\frac{m}{\omega}}\right)}{(r+\phi+\mu m)}\Delta\mu\beta\right]}<0
$$

$$
\frac{du^*}{d\Omega}=\frac{\left[r+\phi+\mu\beta m+\overset{-}{\phi_\psi}\overset{+}{\psi_\Delta}\left(\frac{E}{u}-1\right)\Delta\beta\right]\frac{u}{(r+\phi+\mu m)}\mu\frac{m}{\omega}\frac{q}{p+q}}{-\left[\left(\mu\frac{M}{\omega}+\phi\right)-\left(\overset{-}{\phi_\psi}\overset{+}{\psi_\Delta}(E-u)-\mu(1-\beta)\frac{M}{\Delta\beta}\right)\frac{\left(\overset{>0}{\frac{m}{u}-\frac{m}{\omega}}\right)}{(r+\phi+\mu m)}\Delta\mu\beta\right]}<0
$$

$$
for \quad 0 < 1-\frac{\overset{(-)}{\phi_\psi}}{\phi}\frac{\phi_{\psi_i}}{\phi_{\psi_i\psi_i}} \quad \text{as sufficient condition.}
$$

b) Effects on the rate of utilization of entrepreneurial capacity:

$$
\begin{aligned}
E &= n+u \quad \text{for} \quad E=1 \\
d\Psi &= dn=-du
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\Psi}{dc} &= -\frac{du}{dc}<0, \quad \frac{d\Psi}{d\gamma}=-\frac{du}{d\gamma}<0 \\
\frac{d\Psi}{d\mu} &= -\frac{du}{d\gamma}>0, \quad \frac{d\Psi}{d\Omega}=-\frac{du}{d\Omega}>0
\end{aligned}
$$

c) Effects on the separation rate and the survival rate:

$$\phi = \phi\left(\psi^*\left(\Delta^*\left(x\right),\gamma\right)\right), \quad \text{with} \quad \begin{array}{ll} \phi_{\psi_i} < 0, & \phi_{\psi_i\psi_i} > 0, \\ \psi_{\gamma_i} < 0, & \psi_{\Delta_i} > 0 \end{array}$$

From F we know that $\frac{d\omega}{du} = 1$ and from G we know:

$$0 = \Delta\mu\beta\frac{m}{\omega}d\omega - \beta\Delta\mu\frac{m}{u}du + \left(\left(r+\phi+\mu m\right)\right)d\Delta$$

$$\frac{d\Delta}{du} = \frac{\Delta\mu\beta}{\left(r+\phi+\mu m\right)}\left(\frac{m}{u}-\frac{m}{\omega}\right) > 0$$

$$\begin{aligned}
\frac{d\phi^*}{dc} &= \overset{(-)}{\frac{\partial\phi}{\partial\psi^*}}\overset{(+)}{\frac{\partial\psi^*}{\partial\Delta^*}}\overset{(+)}{\frac{d\Delta}{du}}\overset{(+)}{\frac{du^*}{dc}} < 0, \quad \frac{d\lambda}{dc} = -\frac{d\phi^*}{dc} > 0 \\
\frac{d\phi^*}{d\gamma} &= \overset{(-)}{\frac{\partial\phi}{\partial\psi^*}}\overset{(-)}{\frac{\partial\psi^*}{\partial\gamma}} > 0, \quad \frac{d\lambda}{d\gamma} = -\frac{d\phi^*}{d\gamma} < 0 \\
\frac{d\phi^*}{d\mu} &= \overset{(-)}{\frac{\partial\phi}{\partial\psi^*}}\overset{(+)}{\frac{\partial\psi^*}{\partial\Delta^*}}\overset{(+)}{\frac{d\Delta}{du}}\overset{(-)}{\frac{du^*}{d\mu}} > 0, \quad \frac{d\lambda}{d\mu} = -\frac{d\phi^*}{d\mu} < 0 \\
\frac{d\phi^*}{d\Omega} &= \overset{(-)}{\frac{\partial\phi}{\partial\psi^*}}\overset{(+)}{\frac{\partial\psi^*}{\partial\Delta^*}}\overset{(+)}{\frac{d\Delta}{du}}\overset{(-)}{\frac{du^*}{d\Omega}} > 0, \quad \frac{d\lambda}{d\Omega} = -\frac{d\phi^*}{d\Omega} < 0
\end{aligned}$$

d) Effects on the matching rate, as the percentage of newly started firms:

$$\varepsilon = \frac{\mathcal{M}}{n}$$

Under stationary conditions ($\dot{n} = 0$) $\mathcal{M} = \phi n$ and hence $\varepsilon = \phi$. Therefore,

$$\frac{d\varepsilon^*}{dc} = \frac{d\phi^*}{dc} < 0, \quad \frac{d\varepsilon^*}{d\gamma} = \frac{d\phi^*}{d\gamma} > 0, \quad \frac{d\varepsilon^*}{d\mu} = \frac{d\phi^*}{d\mu} > 0,$$

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